Spin Correlation Phenomena in the Reaction $\bar{N}_a+N_b \rightarrow \bar{Y}_c+Y_d$†

LOYAL DURAND, III, AND JACK SANDWEISS
Yale University, New Haven, Connecticut
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Reactions of the type $\bar{N}_a+N_b \rightarrow \bar{Y}_c+Y_d$, where N_a and N_b are nucleons, and Y_c and Y_d are hyperons, occur with significant probabilities in the interactions of energetic antinucleons. Particular examples which have been studied recently are the reactions $\bar{p}+p \rightarrow \bar{\Lambda}+\Lambda$, $\bar{\Lambda}+\Sigma^0$, $\bar{\Sigma}^0+\Lambda$, $\bar{\Sigma}^-+\Sigma^+$. Pais has discussed some consequences for the reaction cross sections and the polarizations of the final particles of the presumed invariance of the relevant strong interactions under the parity and charge-conjugation operations. In the present paper, these considerations are extended to encompass two-particle spin correlations in the $\bar{Y}_c$, Y_d system. Measurements of the correlation parameters would provide tests for charge conjugation, parity, and *CP* invariance in the strong interactions of strange particles, and could, in addition, be used to check the relation between antihyperon and hyperon decay asymmetry parameters predicted on *CPT* and *T* invariance for the weak interactions, that $\alpha_{\bar{Y}}=-\alpha_Y$. Moreover, measurements of the spin correlation parameters would provide valuable information about the spin dependence of the reactions, hence, some tests for the models have been proposed for the reaction mechanism. Calculations by Bessis, Itzykson, and Jacob, and by Sopkovich using specific models suggest that the correlation parameters may be measurably large. We consider finally in an Appendix the relation between the Wolfenstein-Ashkin spin transition matrix which is used in the body of the paper, and the relativistic parametrization of the transition amplitude in the helicity representation. The general partial-wave expansions of the coefficient functions in the transition matrix are derived.

I. INTRODUCTION

REACTIONS of the type

$$\bar{N}_a+N_b \rightarrow \bar{Y}_c+Y_d, \tag{1}$$

where N_a and N_b are nucleons, and Y_c and Y_d are hyperons, occur with significant probabilities in the interactions of energetic antinucleons.[1,2] Particular examples which have been studied recently include the reactions $\bar{p}+p \rightarrow \bar{\Lambda}+\Lambda$, $\bar{p}+p \rightarrow \bar{\Lambda}+\Sigma^0$, $\bar{p}+p \rightarrow \bar{\Sigma}^0+\Lambda$, and $\bar{p}+p \rightarrow \bar{\Sigma}^-+\Sigma^+$. The strong interactions are presumably invariant under the parity and charge-conjugation interactions, and under the combined operation *CP*. Some consequences of these symmetry operations for the reaction cross sections and the polarizations of the final particles in reactions (1) have been discussed by Pais.[3] It is the purpose of this paper to extend those considerations to the two-particle spin correlations in the $\bar{Y}_c$, Y_d system. Because of the parity violations in the decays of the hyperons, and the concomitant spin-dependent decay asymmetries, the two-particle spin correlation parameters can be determined from the angular correlations in the over-all decay distributions. The reactions are therefore essentially self-analyzing, and it is not necessary to perform second scattering experiments with the decay products.

The spin dependence of reaction (1) is considered in Sec. II using the spin transition matrix methods of Wolfenstein and Ashkin.[4] The cases of even and odd relative (Y_c,Y_d) parity are both considered; the results are quite general,[4a] and could be applied, for example, to reactions involving spin $-\frac{1}{2}$ hyperon isobars as well as to the reactions previously noted. The most general form of the transition matrix which is invariant under the parity operation involves eight independent functions of the scattering angle for either even or odd relative (Y_c,Y_d) parity. The operation of charge conjugation relates the transition matrix for reaction (1) to that for the charge conjugate reaction, $\bar{N}_b+N_a \rightarrow \bar{Y}_d+Y_c$. In the special case of a self-charge-conjugate reaction, $\bar{N}+N \rightarrow \bar{Y}+Y$, the transition matrix involves only six functions of $\cos\theta$. The general forms for the production and decay angular distributions are easily derived; the latter may be expressed in terms of the polarization and spin correlation parameters for the $\bar{Y}_c$, Y_d system.

These general results have several interesting applications. For example, a failure of any of the equalities $I_0(W,\cos\theta)=\bar{I}_0(W,\cos\theta)$, $P_c=P_{\bar{c}}$, $P_d=P_{\bar{d}}$, $C_{ij}=\bar{C}_{ji}$ which relate the differential cross section, polarizations, and spin correlation parameters for a reaction and its

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[1] C. Baltay, E. C. Fowler, J. Sandweiss, J. R. Sanford, H. D. Taft, B. B. Culwick, W. B. Fowler, J. K. Kopp, R. I. Louttit, R. P. Shutt, A. M. Thorndike, and M. S. Webster, *1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 233; C. Baltay, J. Sandweiss, H. Taft, B. B. Culwick, W. B. Fowler, J. K. Kopp, R. I. Louttit, J. R. Sanford, R. P. Shutt, A. M. Thorndike, and M. S. Webster, Bull. Am. Phys. Soc. **8**, 21 (1963); C. Baltay, Yale dissertation, 1963 (unpublished).

[2] R. Armenteros, E. Fett, B. French, L. Montanet, V. Kokitin, M. Szeptcka, Ch. Peyrou, R. Bock, A. Shapira, J. Badier, L. Blaskovicz, A. Equer, B. Gregory, F. Muller, S. J. Goldsack, D. H. Miller, C. C. Butler, B. Tallini, J. Kinson, L. Riddiford, A. Leveque, J. Meyer, A. Verglas, and S. Zylberach, *1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 236.

[3] A. Pais, Phys. Rev. Letters **3**, 242 (1959).

[4] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952); also L. Wolfenstein, in *Annual Reviews of Nuclear Science* (Annual Reviews Inc., Stanford, California, 1956), Vol. 6.

[4a] *Note added in proof*. Spin correlation phenomena have been considered in detail by S. Barshay [Phys. Rev. **113**, 349 (1959)] for the special case in which the final state is restricted to orbital angular momentum zero (production at threshold). Correlation phenomena have been considered for particular models by several authors (Refs. 5–7).

charge conjugate would evince a violation of CP in the reactions. In addition, a nonzero polarization for either final particle along a direction lying in the reaction plane, or a nonvanishing two-particle spin correlation with respect to the normal to the reaction plane and a direction in that plane, would signify a violation of parity. If invariance of the strong interactions under C and P is assumed, the results may be used to test the symmetry properties of the hyperon decay processes. The assumption of CPT and T invariance for the weak interactions leads to the prediction, as yet untested, that the asymmetry parameters for the decay of a hyperon and its antiparticle are related by $\alpha_{\bar{Y}}=-\alpha_Y$. As was noted by Pais,[3] the equality of the polarizations of the hyperons produced at the angles θ, ϕ, and of the antihyperons produced in the charge-conjugate reaction at the angles $\pi-\theta$, $\pi+\phi$, permits a direct test of this a prediction. However, the absence of significant polarizations does not preclude at least a partial test of this relation if any of the two-particle spin correlation parameters are nonzero. The angular asymmetries in the decay distribution may also be used to provide lower bounds on the asymmetry parameters. More conventionally, measurements of the spin correlation parameters would provide valuable information about the spin dependence of the reactions, and can be used to test various models which have been proposed for the reaction mechanism. For example, quite different predictions for the correlation parameters are obtained from the single K^* exchange model proposed by Bessis, Itzykson, and Jacob,[5] the modified K^* plus K exchange model of Sopkovich,[6] and the K^* Regge pole model of Chan.[7] Detailed calculations based on these models suggest that the spin correlation parameters may be measurably large.

We consider finally in an Appendix the relation between the Wolfenstein-Ashkin spin transition matrix M,[4] and the relativistic transition matrix as parametrized using the helicity representation for angular momentum.[8] It is shown in particular that the M-matrix approach is correct relativistically if all spin expectation values are referred to the respective rest systems of the particles in question. The general partial-wave expansions of the coefficient functions in the transition matrix are also derived, and the appearance of certain angular factors in the spin polarization and correlation coefficients is demonstrated.

II. SPIN CORRELATIONS IN THE REACTION $\bar{N}_a+N_b \to \bar{Y}_c+Y_d$

a. General Properties of the Transition Matrix

The spin-dependent features of a reaction $\bar{a}+b\to\bar{c}+d$ involving only spin-$\frac{1}{2}$ particles can be described using a 4×4 transition matrix M which expresses the spin dependence of the final wave function in terms of that of the initial wave function. In particular, if the density matrix of the initial system is $\rho_{\bar{a}b}$, that of the final system is given by

$$\rho_{\bar{c}d}=M\rho_{\bar{a}b}M^\dagger. \tag{2}$$

If we assume, furthermore, that $\rho_{\bar{a}b}$ is normalized,

$$\mathrm{Tr}\rho_{\bar{a}b}=1,$$

and that the elements of M are properly normalized transition amplitudes referred to the center-of-mass system, the differential reaction cross section in that system is given by

$$d\sigma/d\Omega=\mathrm{Tr}\rho_{\bar{c}d}=\mathrm{Tr}M\rho_{\bar{a}b}M^\dagger, \tag{3}$$

where the trace extends over the spin indices only. More generally, the expectation value of any spin-dependent operator A is given in the final state by

$$\langle A\rangle\,\mathrm{Tr}\rho_{\bar{c}d}=\mathrm{Tr}A\rho_{\bar{c}d}. \tag{4}$$

The 4×4 matrices M and ρ may be expressed conveniently as linear combinations of the sixteen independent matrices σ_{ij}, $i,j=0, 1, 2, 3$, formed by taking direct products of 2×2 matrices,

$$\sigma_{ij}=\sigma_{1i}\sigma_{2j}.$$

Here, σ_0 is the 2×2 unit matrix, and the σ_j with $j=1, 2, 3$ are the usual Pauli spin matrices. The σ_{ij} satisfy the orthogonality relations

$$\mathrm{Tr}\sigma_{ij}\sigma_{kl}=4\delta_{ik}\delta_{jl}.$$

Since these matrices span the space of 4×4 matrices, M and ρ may be written in the form

$$M=\sum_{ij} m_{ij}\sigma_{ij}, \quad \rho=\sum_{ij}\rho_{ij}\sigma_{ij}, \tag{5}$$

where

$$m_{ij}=\tfrac{1}{4}\,\mathrm{Tr}\sigma_{ij}M$$

and

$$\rho_{ij}=\tfrac{1}{4}\,\mathrm{Tr}\sigma_{ij}\rho,\ \rho_{ij}^*=\rho_{ij}.$$

It is well known in the case of nonrelativistic elastic scattering,[4] that the matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, which appear in the expressions for the density matrix, can be interpreted as the Pauli spin matrices for the initial or final particles. This interpretation is also valid for inelastic processes and for relativistic particles provided, in the latter case, that the spin operators are assumed to refer to the rest systems of the respective particles. This point is discussed in more detail in Appendix B. We will henceforth follow the convention that the operators $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ act, respectively, on the spin indices of the antiparticles and the particles.

The invariance of the strong interactions under proper inhomogeneous Lorentz transformations implies that we need consider M only in the center-of-mass system, and that it must transform in that system as a

[5] D. Bessis, C. Itzykson, and M. Jacob, Nuovo Cimento **27**, 376 (1963).
[6] N. J. Sopkovich, dissertation, Carnegie Institute of Technology, 1962 (unpublished).
[7] C. H. Chan, Phys. Rev. **133**, B431 (1964).
[8] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

TABLE I. Transformation properties of the rotational invariants which can enter the general transformation matrix $M(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{l},\mathbf{m},\mathbf{n})$.

Invariant	Signature, P	Signature, C	Signature, CP
1	+	+	+
$(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}$	+	+	+
$(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}$	+	−	−
$\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{l}$	+	+	+
$\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{n}$	+	+	+
$\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}$	+	+	+
$\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{m}+\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{l}$	+	+	+
$\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{m}-\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{l}$	+	−	−
$(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{l}$	−	−	+
$(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{l}$	−	+	−
$(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{m}$	−	−	+
$(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{m}$	−	+	−
$\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{n}+\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{l}$	−	−	+
$\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{n}-\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{l}$	−	+	−
$\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{n}+\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{m}$	−	−	+
$\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{n}-\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{m}$	−	+	−

scalar under rotations. The sixteen independent components of M must therefore be expressed in terms of scalar products formed from the initial and final momenta, and the Pauli spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$. The discussion is greatly facilitated by the introduction of three orthogonal vectors which define a right-handed coordinate system in the center-of-mass frame,[9]

$$\mathbf{l}=\hat{p}'+\hat{p}=2\hat{l}\cos\tfrac{1}{2}\theta\,,\quad \mathbf{m}=\hat{p}'-\hat{p}=2\hat{m}\sin\tfrac{1}{2}\theta\,,\quad \mathbf{n}=\hat{p}\times\hat{p}'=\hat{n}\sin\theta\,. \tag{6}$$

Here, $\hat{p}$ and $\hat{p}'$ are unit vectors in the directions of motion of the incident and emergent antiparticles; the scattering angle in the center-of-mass system is defined by

$$\cos\theta=\hat{p}\cdot\hat{p}'=\hat{p}(\bar{N}_a)\cdot\hat{p}(\bar{Y}_c)\,. \tag{7}$$

A complete set of sixteen independent rotational scalars constructed from these vectors and the Pauli matrices is given in Table I. These operators may appear in the general expression for the transition matrix multiplied by complex functions of the scattering angle and the total energy in the center-of-mass system. The invariance of the strong interactions under space inversions requires that M transform as a true scalar (pseudoscalar) if the relative parity of the final particles is the same as (different from) that of the initial particles. Under the parity transformation $\boldsymbol{\sigma}\to\boldsymbol{\sigma}$, $\mathbf{l}\to-\mathbf{l}$, $\mathbf{m}\to-\mathbf{m}$, and $\mathbf{n}\to\mathbf{n}$. Thus, it is necessary that

$$M(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{l},\mathbf{m},\mathbf{n})=\eta_p M(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,-\mathbf{l},-\mathbf{m},\mathbf{n})\,, \tag{8}$$

where η_p is equal to $+1(-1)$ if the relative intrinsic parity of the initial and final particles is even (odd). The charge-conjugation operation changes the antiparticles in reaction (1) into particles with the same spin projections and momenta, and conversely. However, the basic vectors are always defined in terms of the momenta of the antiparticles, and the relabeling of the particles therefore induces the transformation $\mathbf{l}\to-\mathbf{l}$, $\mathbf{m}\to-\mathbf{m}$, $\mathbf{n}\to\mathbf{n}$; in addition, the roles of the antiparticle and particle spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are interchanged, $\boldsymbol{\sigma}_1\to\boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_2\to\boldsymbol{\sigma}_1$. The assumption of charge-conjugation invariance for the strong interactions consequently implies that

$$M(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{l},\mathbf{m},\mathbf{n})=\eta_c M^c(\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_1,-\mathbf{l},-\mathbf{m},\mathbf{n}),\ |\eta_c|=1\,, \tag{9}$$

where M^c is the transition matrix for the charge-conjugate reaction, $\bar{N}_b+N_a\to\bar{Y}_d+Y_c$. We note also the effect of the combined operation CP,

$$M(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{l},\mathbf{m},\mathbf{n})=\eta_{cp} M^c(\boldsymbol{\sigma}_2,\boldsymbol{\sigma}_1,\mathbf{l},\mathbf{m},\mathbf{n}),\ |\eta_{cp}|=1\,. \tag{10}$$

The time reversal invariance of the strong interactions unfortunately does not impose any further useful restrictions on the structure of M: the time inverse reaction, $\bar{Y}_c+Y_d\to\bar{N}_a+N_b$ is not accessible experimentally.

The most general form for the transition matrix that transforms correctly under rotations and reflections is easily determined. For even relative (Y_c,Y_d) parity,[10]

$$M=M_1+M_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+M_3(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}+M_4(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}+M_5\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{l}+M_6\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{m}$$
$$+M_7[\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{n}+\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{l}]+M_8[\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{m}-\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{l}]\,,\quad \eta_p=+1\,. \tag{11}$$

The coefficients M_j are functions of the total energy W and the scalar product $\hat{p}\cdot\hat{p}'=\cos\theta$. Charge-conjugation invariance imposes the additional restriction that the functions M_j be related to the functions $M_j{}^c$ as

$$M_j{}^c(W,\cos\theta)=M_j(W,\cos\theta),\ j\neq4,8\,,\quad M_4{}^c(W,\cos\theta)=-M_4(W,\cos\theta)\,,\quad M_8{}^c(W,\cos\theta)=-M_8(W,\cos\theta)\,. \tag{12}$$

The phase conventions have been chosen so that $\eta_c=1$. It should be noted that θ is always the scattering angle of the antiparticle, and thus refers to different particles in M_j and $M_j{}^c$. In the case of the self-charge-conjugate reactions $\bar{N}+N\to\bar{Y}+Y$, the restrictions noted in Eq. (12) imply that $M_4(W,\cos\theta)=M_8(W,\cos\theta)=0$. These relations follow also from the more general requirement of invariance under the combined operation CP.

The general form for the transition matrix in the case

[9] The particular choice of basis vectors is governed by the requirement that M have simple transformation properties under C and P. Other possible choices, for example, the set $\hat{p}'$, $\hat{n}$, $\hat{n}\times\hat{p}'$, which is natural for the helicity transformation $\mathbf{f}$ of Appendix B, lead to rather complicated relations between the coefficient functions, in this case under C.

[10] We assume, of course, that the (N_a,N_b) relative parity is even, hence, that the $(\bar{N}_a,N_b)$ parity is odd.

of odd relative (Y_c,Y_d) parity is given by

$$M=M_1(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{l}+M_2(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{l}+M_3(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{m}+M_4(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{m}+M_5[\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{n}+\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{l}]$$
$$+M_6(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{m}+M_7[\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{n}+\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2,\mathbf{m}]+M_8(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{l},\quad \eta_p=-1\,, \quad (13)$$

with the additional restrictions, implied by charge-conjugation invariance, that the coefficient functions M_j and M_j^c be related as

$$\begin{aligned} M_j(W,\cos\theta)&=M_j^c(W,\cos\theta),\ j=2,4,6,8\\ M_j(W,\cos\theta)&=-M_j^c(W,\cos\theta),\ j=1,3,5,7.\end{aligned} \quad (14)$$

b. General Results for Spin Correlation Phenomena

The density matrix for the final state in the reaction $\bar{N}_a+N_b\rightarrow\bar{Y}_c+Y_d$ is completely specified by the sixteen coefficients ρ_{ij} in the expansion in Eq. (5). These coefficients may be identified with the usual spin polarization vectors $\mathbf{P}_{\bar{c}}$ and $\mathbf{P}_d$, and the joint spin correlation parameters C_{ij}, i, $j=l$, m, n, defined according to the relations

$$I_0P_{\bar{c}j}=\mathrm{Tr}\sigma_{1j}\rho_{\bar{c}d}\,, \quad (15)$$

$$I_0P_{dj}=\mathrm{Tr}\sigma_{2j}\rho_{\bar{c}d}\,, \quad (16)$$

and

$$I_0C_{ij}=\mathrm{Tr}\sigma_{1i}\sigma_{2j}\rho_{\bar{c}d}\,. \quad (17)$$

Here, the σ_j are the components of $\boldsymbol{\sigma}$ in the $\hat{l}$, $\hat{m}$, and $\hat{n}$ directions, and $I_0(W,\theta,\phi)$ is the differential reaction cross section,

$$I_0(W,\theta,\phi)=\mathrm{Tr}\rho_{\bar{c}d}=\mathrm{Tr}M\rho_{\bar{a}b}M^\dagger. \quad (18)$$

An additional spin parameter C is also of interest,

$$C=C_{ll}+C_{mm}+C_{nn}; \quad (19)$$

this parameter represents the expectation value of $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ in the final state, hence, measures the relative weights of the singlet and triplet spin configurations.

In the situation most likely to be encountered in practice, the initial particles are unpolarized. The density matrix $\rho_{\bar{a}b}$ is then equal to $\frac{1}{4}$ times the 4×4 unit matrix, and $\rho_{\bar{c}d}$ is given by

$$\rho_{\bar{c}d}=\tfrac{1}{4}MM^\dagger. \quad (20)$$

We will confine our attention to this case, although it will become clear that more general situations must be considered if the structure of M is to be determined in full.

A number of general properties of the polarization and spin correlation parameters may be deduced without explicit calculation. It is readily verified that the parameters $P_{\bar{c}j}$, P_{dj}, and C_{ij} are all real and less than unity in absolute value.[11] The joint spin parameter C is also real and is restricted to the range $-3\leq C\leq 1$. The invariance of the strong interactions under the combined operation CP, and the resulting symmetry properties of transition matrix, Eq. (10), lead to several relations between the parameters for the reactions $\bar{N}_a+N_b\rightarrow\bar{Y}_c+Y_d$ and $\bar{N}_b+N_a\rightarrow\bar{Y}_d+Y_c$: (i) The total cross sections and the angular distributions of the antihyperons are identical for the two reactions. (The production angles are defined in terms of the momenta of the antiparticles, different in the two cases.) (ii) The polarization of the antihyperon (hyperon) produced at the angles θ, ϕ in the $\bar{Y}_cY_d$ reaction is the same as that of the hyperon (antihyperon) produced at the angles $\pi-\theta$, $\pi+\phi$ in the charge conjugate $\bar{Y}_dY_c$ reaction. Relations (i) and (ii) have been noted previously by Pais.[3] (iii) The spin correlation parameters C_{ij} and $\bar{C}_{ij}$ for a reaction and its charge conjugate are related by $C_{ij}=\bar{C}_{ji}$. These results may be sharpened somewhat if it is assumed that the strong interactions are invariant under P and C separately. Conservation of parity leads to the familiar restrictions, (iv), $P_{\bar{c}l}=P_{\bar{c}m}=P_{dl}=P_{dm}=0$, and (v), $C_{ln}=C_{nl}=C_{mn}=C_{nm}=0$. Charge-conjugation invariance adds new restrictions only in the case of self-charge-conjugate reactions, for which, (vi), $P_{\bar{Y}}=P_Y$, and $C_{lm}=C_{ml}$. We note finally several general requirements which follow from the rotational invariance of $MM^\dagger$: (vii) For forward production of the antihyperon, it is not possible to distinguish the $\hat{m}$ and $\hat{n}$ directions, but $\hat{l}$ is well defined. Consequently, for $\theta=0$, $P_{\bar{c}}=P_d=0$, $C_{mm}=C_{nn}$, $C_{lm}=C_{ln}=0$, and $C_{ml}=C_{nl}=0$. (viii) For $\theta=\pi$, $\hat{m}$ is well defined, but $\hat{l}$ and $\hat{n}$ are not, and one finds that $P_{\bar{c}}=P_d=0$, $C_{ll}=C_{nn}$, $C_{lm}=C_{nm}=0$, and $C_{ml}=C_{mn}=0$. In stating relations (vii) and (viii), we have assumed that parity is conserved. The results for the differential cross section, polarization, and spin correlation parameters specific to the cases of even and odd relative (Y_c,Y_d) parity are given in Appendix A. It is not possible to distinguish the two cases in general; however, particular models for the reaction mechanism, for example, single-particle exchange, may lead to quite different predictions for the spin correlation parameters for the different relative parities.[5–7]

The violation of parity in the decays of the hyperons and antihyperons, with the concomitant dependence of the decay angular distributions on the particle polarizations, provides a powerful tool for the analysis of spin correlations in reactions which lead to $\bar{\Lambda}\Lambda$, $\bar{\Sigma}^-\Sigma^+$, $\bar{\Lambda}\Sigma^+$, $\bar{\Sigma}^-\Lambda$, or $\bar{\Xi}^+\Xi^-$ final states. Spin correlations in systems which involve Σ^0 or $\bar{\Sigma}^0$ hyperons can be analyzed using the asymmetry in the decay of the Λ or $\bar{\Lambda}$ produced in the fast electromagnetic transition $\Sigma^0\rightarrow\Lambda+\gamma$ or

[11] The reality of the parameters P_j and C_{ij} follows from the Hermitian character of the density matrix and the operators σ_{ij}. The bounds on the absolute values are readily established by applying the Schwartz inequality to the expectation values of the operators $(1+\sigma_{ij})^2$.

$\bar{\Sigma}^0 \to \bar{\Lambda}+\gamma$, but with some loss in sensitivity resulting from the loss of polarization in the intermediate step. Because of the absence of any significant decay asymmetries, it does not appear feasible at the present time to measure spin correlations in two-particle systems which involve a $\bar{\Sigma}^+$ or Σ^- hyperon. We shall therefore restrict our attention to those cases for which a decay asymmetry is expected, and calculate the resulting spin-dependent correlations in the angular distributions of the decay nucleon and antinucleon.

The density matrix $\rho_{\bar{c}d}=\frac{1}{4}MM^\dagger$ specifies the probability that the antihyperons produced in the θ, ϕ direction in the center-of-mass system, and the hyperon produced in the $\pi-\theta$, $\pi+\phi$ direction, be found in definite spin states when observed in their respective rest systems.[12] The subsequent decays of those particles are described in their rest systems by the transition matrices[13]

$$M_{\bar{c}}=A_{\bar{c}}+B_{\bar{c}}\boldsymbol{\sigma}_1\cdot\hat{p}_1 \tag{21}$$

and

$$M_d=A_d+B_d\boldsymbol{\sigma}_2\cdot\hat{p}_2\,, \tag{22}$$

where $\hat{p}_1$ and $\hat{p}_2$ are unit vectors in the directions of motion of the resultant antibaryon and baryon, respectively.[14] The coefficients A and B are the $J=\frac{1}{2}$ S- and P-wave decay amplitudes, normalized according to the relation[15]

$$|A|^2+|B|^2=(4\pi)^{-1}. \tag{23}$$

The production of the hyperons, and their decay as seen in their respective rest systems, is therefore completely described by the density matrix

$$\rho=M_{\bar{c}}M_d\rho_{\bar{c}d}M_d^\dagger M_{\bar{c}}^\dagger. \tag{24}$$

Because it is not feasible at present to measure such quantities as the polarizations of the decay nucleons, we will consider only the angular distribution of those particles.

The cross section for producing $\bar{Y}_c$ and Y_d in a $\bar{N}_a,N_b$ collision, with $\bar{Y}_c$ emerging in the center-of-mass system in the θ, ϕ direction in an element of solid angle $d\Omega$, and the decay antibaryon and baryon emerging in $d\Omega_1$ and $d\Omega_2$ in the rest systems of $\bar{Y}_c$ and Y_d, respectively, is given by $\mathrm{Tr}\rho$,

$$\begin{aligned} d^3\sigma/(d\Omega d\Omega_1 d\Omega_2)&=I(W,\theta,\phi,\theta_1,\phi_1,\theta_2,\phi_2)\\ &=\mathrm{Tr}M_{\bar{c}}^\dagger M_{\bar{c}}M_d^\dagger M_d\rho_{\bar{c}d}. \end{aligned} \tag{25}$$

This expression may be simplified using Eqs. (21), (22), and (23).

$$M_{\bar{c}}^\dagger M_c=(4\pi)^{-1}[1+\alpha_{\bar{c}}\boldsymbol{\sigma}_1\cdot\hat{p}_1]\,, \tag{26}$$

and

$$M_d^\dagger M_d=(4\pi)^{-1}[1+\alpha_d\boldsymbol{\sigma}_2\cdot\hat{p}_2]\,, \tag{27}$$

where, in each case, α is the asymmetry parameter for the decay[14]

$$\alpha=2\,\mathrm{Re}A^*B/[|A|^2+|B|^2]\,. \tag{28}$$

Using these results, the expression for the cross section may be written in the form

$$\begin{aligned} I&=(4\pi)^{-2}\,\mathrm{Tr}(1+\alpha_{\bar{c}}\boldsymbol{\sigma}_1\cdot\hat{p}_1)(1+\alpha_d\boldsymbol{\sigma}_2\cdot\hat{p}_2)\rho_{\bar{c}d}\\ &=(I_0/16\pi^2)[1+\alpha_{\bar{c}}\mathbf{P}_{\bar{c}}\cdot\hat{p}_1+\alpha_d\mathbf{P}_d\cdot\hat{p}_2\\ &\qquad+\alpha_{\bar{c}}\alpha_d\sum_{i,j=l,m,n}C_{ij}\hat{p}_{1,i}\hat{p}_{2,j}]\,. \end{aligned} \tag{29}$$

This equation displays clearly the remarkable fact that, as a consequence of the parity violation in the decays of the hyperons, the complete set of polarization and spin correlation parameters for the reaction can be determined from observations of the asymmetries in the decay angular distributions, and the angular correlations of the decay momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. Measurements of the reaction cross section $I_0(W,\cos\theta)$, the hyperon and antihyperon polarizations, and the five independent, nonvanishing spin correlation parameters, provide eight restrictions on the eight complex amplitude functions $M_j(W,\cos\theta)$ for given values of W and θ. Fifteen independent quantities are necessary to determine these functions up to an over-all phase. The extra conditions on the M_j can probably be obtained from studies of the asymmetries and angular correlations in an experiment in which the incident antinucleon or the target nucleon is polarized, but this has not been checked in detail. The case of self-charge-conjugate reactions is somewhat more favorable, since the functions M_4 and M_8 in Eq. (11) must vanish. The number of independent parameters in the transition matrix is consequently reduced by four, while the number of independent observables is only reduced by two $[P_{\bar{c}}=P_d,\ C_{lm}=C_{ml}]$.

The analysis of the reactions $\bar{p}+p\to\bar{\Lambda}+\Sigma^0$ and $\bar{p}+p\to\bar{\Sigma}^0+\Lambda$ is complicated both experimentally and theoretically by the rapid, parity conserving, electromagnetic decay of the Σ^0 or $\bar{\Sigma}^0$. For simplicity, we will restrict our attention to the first reaction. The density matrix ρ, which describes the production of the $\bar{\Lambda},\Sigma^0$ system, decay of the Σ^0, $\Sigma^0\to\Lambda+\gamma$, and the subsequent decays of the Λ and $\bar{\Lambda}$ hyperons, is given by

$$\rho=M_{\bar{\Lambda}}M_\Lambda M_\Sigma\rho_{\bar{\Lambda}\Sigma}M_\Sigma^\dagger M_\Lambda^\dagger M_{\bar{\Lambda}}^\dagger\,, \tag{30}$$

where

$$\rho_{\bar{\Lambda}\Sigma}=M\rho_{\bar{p}p}M^\dagger\to\tfrac{1}{4}MM^\dagger \tag{31}$$

for unpolarized initial particles. The transition matrix M_Σ, which describes the decay of the Σ^0, is given with an appropriate normalization by

$$M_\Sigma=(8\pi)^{-1/2}\boldsymbol{\sigma}\cdot\hat{p}_\Lambda\times\boldsymbol{\varepsilon} \tag{32}$$

[12] This restriction is discussed in detail in Appendix B.

[13] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1645 (1957); J. Leitner, Nuovo Cimento 8, 68 (1958).

[14] The convention used in Eqs. (21) and (22) follows that of recent experimental papers in using the direction of motion of the nucleon rather than that of the pion in the $\boldsymbol{\sigma}\cdot\hat{p}$ term. With this convention, the asymmetry parameter in the decay of a polarized hyperon is equal to the helicity of the nucleon in the decay of an unpolarized hyperon.

[15] This normalization is appropriate to a situation in which both hyperons are observed to decay. Only such events are useful in determining the two-particle spin correlations.

for even relative (Σ^0,Λ) parity. In the unlikely circumstance that the relative (Σ^0,Λ) parity should be odd,[16] the decay matrix would be given by

$$M_\Sigma=(8\pi)^{-1/2}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}. \tag{33}$$

Here $\boldsymbol{\varepsilon}$ is the polarization vector of the photon, $\boldsymbol{\varepsilon}\cdot\hat{p}_\Lambda=0$, and $\hat{p}_\Lambda$ is a unit vector in the direction of motion of the decay Λ as seen in the rest system of the Σ^0. The most general result for the correlated production and decay angular distributions is given by $\mathrm{Tr}\rho$,

$$\begin{aligned}I(\bar{\Lambda},\Sigma^0)&=d^4\sigma/(d\Omega d\Omega_\Lambda d\Omega_1 d\Omega_2)\\&=\mathrm{Tr}M_{\bar{\Lambda}}{}^\dagger M_{\bar{\Lambda}}M_\Sigma{}^\dagger M_\Lambda{}^\dagger M_\Lambda M_\Sigma\rho_{\bar{\Lambda}\Sigma},\end{aligned} \tag{34}$$

where the elements of solid angle refer successively to the direction of motion of the $\bar{\Lambda}$ as seen in the center-of-mass system, and to the directions of the Λ, antinucleon, and nucleon in the decays of the Σ^0, $\bar{\Lambda}$, and Λ, each referred to the rest system of the decaying particle. The trace includes a sum over the photon polarizations. The products of decay matrices are easily reduced, and after summing over the photon polarizations, one obtains

$$\begin{aligned}I(\bar{\Lambda},\Sigma^0)&=(4\pi)^{-3}\,\mathrm{Tr}(1+\alpha_{\bar{\Lambda}}\boldsymbol{\sigma}_1\cdot\hat{p}_1)\\&\qquad\times(1-\alpha_\Lambda\hat{p}_2\cdot\hat{p}_\Lambda\boldsymbol{\sigma}_2\cdot\hat{p}_\Lambda)\rho_{\bar{\Lambda}\Sigma}\\&=(I_0/64\pi^3)[1+\alpha_{\bar{\Lambda}}\mathbf{P}_{\bar{\Lambda}}\cdot\hat{p}_1-\alpha_\Lambda\hat{p}_2\cdot\hat{p}_\Lambda\hat{p}_\Lambda\cdot\mathbf{P}_\Sigma\\&\qquad-\alpha_{\bar{\Lambda}}\alpha_\Lambda\hat{p}_2\cdot\hat{p}_\Lambda\sum_{ij=l,m,n}C_{ij}\hat{p}_{1i}\hat{p}_{\Lambda j}],\end{aligned} \tag{35}$$

irrespective of the relative (Σ^0,Λ) parity. The nonvanishing polarization and spin correlation parameters can again be determined by measuring the asymmetries in the angular distribution, and the correlations between the directions of motion of the decay nucleon, antinucleon, and the intermediate Λ. The expression for $I(\bar{\Lambda},\Sigma^0)$ may be simplified, but with some loss of information, by integrating over the direction of motion $\hat{p}_{\bar{\Lambda}}$ of the Λ in the $\Sigma^0\to\Lambda+\gamma$ decay; this results in an expression of the form given in Eq. (29), but with $\alpha_{\bar{c}}=\alpha_{\bar{\Lambda}}$ and $\alpha_d=-\frac{1}{3}\alpha_\Lambda$.

c. Applications of Spin Correlation Phenomena

The assumption of CPT and T invariance for the weak interactions leads to the prediction, as yet untested, that the asymmetry parameter for the decay of an antihyperon is the negative of that for the corresponding hyperon, $\alpha_{\bar{Y}}=-\alpha_Y$. The equality of $P_{\bar{Y}}$ and P_Y in the self-charge-conjugate reactions $\bar{p}+p\to\bar{\Lambda}+\Lambda$, $\bar{p}+p\to\bar{\Sigma}^-+\Sigma^+$, and $\bar{p}+p\to\bar{\Xi}^++\Xi^-$ permits a direct test of this relation provided the polarizations are large enough to give a significant decay asymmetry.[17] The equality of the polarizations of the hyperon produced at the angles θ,ϕ in a given reaction, and the antihyperon produced at the angle $\pi-\theta$, $\pi+\phi$ in the charge-conjugate reaction, permits a second independent test of the equality $\alpha_{\bar{Y}}=-\alpha_Y$ using general reactions $\bar{N}_a+N_b\to\bar{Y}_d+Y_c$. The absence of significant polarizations, to be expected on the basis of single-particle exchange models for reactions of this type does not preclude the study of $\alpha_{\bar{Y}}$ if the angular correlation parameters $\alpha_{\bar{c}}\alpha_d C_{ij}$ for the two-particle decay distribution are measurably large. The equalities $C_{ij}=\bar{C}_{ji}$, which relate the spin correlation parameters in a reaction and its charge conjugate, permit a direct comparison of the products $\alpha_{\bar{c}}\alpha_d$ and $\alpha_c\alpha_{\bar{d}}$; these should be equal. If, say, $\alpha_{\bar{c}}$ is known to be equal to $-\alpha_c$ from some independent experiment, a direct comparison of α_d and $\alpha_{\bar{d}}$ is possible. There are in general five possible experiments of this type, corresponding to the five nonvanishing correlation coefficients. We note also that, if the angular correlations are measurably large, the limits $|C_{ij}|\leq 1$ provide lower bounds for the products $|\alpha_{\bar{c}}\alpha_d|$, hence, if one of the asymmetry parameters is known, lower bounds for the absolute value of the second parameter. In the case of self-charge-conjugate reactions, assuming that $\alpha_{\bar{Y}}=-\alpha_Y$, one obtains a lower bound on $|\alpha_Y|^2$. These bounds could, in principle, be more sensitive than those obtained from the values of $|\alpha P|$. The argument may be inverted by using the upper bound $|\alpha_{\bar{c}}\alpha_d|=1$ in conjunction with measured angular correlation parameters to obtain a lower bound for $|C_{ij}|$. In the special case that the joint correlation parameter C exceeds unity in absolute value, as determined using either the foregoing bound, or known values for the product $|\alpha_{\bar{c}}\alpha_d|$, C must lie in the range $-3\leq C<-1$. The sign of the product $\alpha_{\bar{c}}\alpha_d$ is then the negative of the sign of $\alpha_{\bar{c}}\alpha_d C$ determined from the decay angular correlations.

One may also use polarization and spin correlation phenomena to test for possible violations of CP, P, or C in the strong interactions involving strange particles. Possible tests are as follows: (i) A failure of any of the

[16] Even (Σ,Λ) relative parity is strongly favored by the invariant mass spectrum of Dalitz pairs in the decay $\Sigma^0\to\Lambda^0+e^++e^-$ [H. Courant, H. Filthuth, P. Franzini, R. G. Glasser, A. Minguzzi-Ranzi, A. Segar, W. Willis, R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Sakitt, B. Sechi-Zorn, N. Seeman, and G. A. Snow, Phys. Rev. Letters **10**, 409 (1963)]. In addition, odd $K\Sigma N$ parity is favored by the data of R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **8**, 175 (1962), while odd $K\Lambda N$ parity, hence, even $\Sigma\Lambda$ parity is favored by the experiment of Block *et al.* [M. M. Block, E. B. Brucker, J. S. Hughes, T. Kokucki, C. Meltzer, F. Anderson, A. Pevsner, E. M. Harth, J. Leitner, and H. O. Cohn, *ibid.* **3**, 291 (1959)].

[17] Present values of the asymmetry parameters for hyperon decays are as follows: $\Lambda\to p+\pi^-$, $\alpha_\Lambda=0.62\pm0.07$ [J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963)]; $\Sigma^+\to p'+\pi^0$, $\alpha_{\Sigma^0}=-0.79_{-0.08}^{+0.09}$ [R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **9**, 66 (1962); E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, *ibid.* **8**, 75 (1962); B. Cork, L. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. **120**, 1000 (1960)]; $\Xi^-\to\Lambda+\pi^-$, $\alpha_{\Xi^-}=-0.50\pm0.13$ [L. W. Alvarez, J. P. Berg, R. Kalbsfeisch, J. Button-Shafer, F. T. Solmitz, M. L. Stevenson, and H. K. Ticho, *1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 433], and $\alpha_{\Xi^-}=-1.0_{+0.35}^{-0.0}$ [L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, L. Gray, J. Leitner, S. Lichtman, and J. Westgard, *ibid.*, p. 437]. The convention for the sign of α is in each case that noted in footnote 14.

equalities $I_0=\bar{I}_0$, $P_{\bar{c}}=P_c$, $P_d=P_{\bar{d}}$, $C_{ij}=\bar{C}_{ji}$ which relate the differential cross sections, polarizations, and spin correlation parameters for a reaction and its charge conjugate would evince a violation of CP. (ii) A nonzero value of any of the parameters $P_{\bar{c}l}$, $P_{\bar{c}m}$, P_{dl}, P_{dm}, C_{ln}, C_{nl}, C_{nm}, C_{mn} would indicate a violation of P. There do not appear to be any tests for C invariance which do not at the same time test CP or P.

In addition to the foregoing, relatively simple applications, the study of spin correlation phenomena provides a useful, or indeed, essential, adjunct to the determination of possible mechanisms for the reaction $\bar{N}_a+N_b\rightarrow\bar{Y}_c+Y_d$. Although it has been seen that a complete evaluation of the transition matrix for the reaction is not possible on the basis of only those measurements that we have discussed, much useful information may nevertheless be obtained from the comparison of the predictions of specific models with the experimental results. The relevance of spin correlation phenomena to tests of the single particle exchange model for the reactions $\bar{p}+p\rightarrow\bar{\Lambda}+\Lambda$, $\bar{\Lambda}+\Sigma^0$, $\bar{\Sigma}^0+\Lambda$, and $\bar{\Sigma}^-+\Sigma^+$, has been emphasized by Bessis *et al.*[5] The experimental cross sections for the $\bar{\Lambda}\Lambda$, $\bar{\Lambda}\Sigma^0$, and $\bar{\Sigma}^0\Lambda$ processes are strongly peaked in the forward direction at high energies; the data for the $\bar{\Sigma}^-\Sigma^+$ reaction are less conclusive, but some peaking may be indicated. The striking tendency for the antiparticle to maintain its original direction of motion is suggestive of a long-range exchange mechanism for the reaction. Although it has been argued[5,18] that a single K-meson exchange mechanism is unable to account for the angular distribution in the $\bar{\Lambda}\Lambda$ reaction, assuming the K meson to be pseudoscalar relative to the ΛN system, this conclusion is incorrect: For reasonable values of the $K\Lambda N$ coupling constant, the S-wave transition amplitudes exceed the limits imposed by unitarity. Reduction of these amplitudes to the unitarity limits results in a reasonable angular distribution. Alternative models for the $\bar{\Lambda}\Lambda$ reaction based on the exchange of a single $J=1^-K^*$ meson (885 MeV $K-\pi$ resonant state[19]) have been studied by Bessis *et al.*,[5] Sopkovich,[6] Chan,[7] and Watson.[18] These models also lead to reasonable angular distributions for the production process. In addition, the exchange of a spin-1 particle can lead to nonzero spin correlations in final state. The joint correlation parameter C was calculated by Bessis *et al.*[5] for the $\bar{\Lambda}\Lambda$, $\bar{\Lambda}\Sigma^0$, $\bar{\Sigma}^0\Lambda$, and $\bar{\Sigma}^-\Sigma^+$ processes for an incident momentum of 3 BeV/c. This parameter was found to be small in the angular region in which the cross section is large, and would consequently be difficult to measure. What is perhaps a more realistic model was considered by Sopkovich,[6] who modified the single K^* exchange mechanism by including single K-meson exchange, and, in addition, some absorptive effects. The latter were calculated using an optical potential matched to the $\bar{p}p$ elastic scattering cross sections. This model predicts measurably large values of C_{mm} and C_{nn} at forward angles for the reaction $\bar{p}+p\rightarrow\bar{\Lambda}+\Lambda$ at 3.3 BeV/c incident antiproton momentum.[20] The correlation parameters were not calculated for $\bar{\Lambda}\Sigma^0$, $\bar{\Sigma}^0\Lambda$, and $\bar{\Sigma}^-\Sigma^+$ reactions, but should not differ greatly from those calculated for the $\bar{\Lambda}\Lambda$ reaction. Measurements of the hyperon and antihyperon polarization and spin correlation parameters for these reactions would be of great interest in themselves, and would also serve as a useful check on the validity of the various single particle exchange models.

APPENDIX A: RESULTS FOR THE REACTION CROSS SECTIONS, POLARIZATIONS, AND SPIN CORRELATION PARAMETERS

The results for the differential cross section, and the polarization, and spin correlation parameters for the reaction $\bar{N}_a+N_b\rightarrow\bar{Y}_c+Y_d$ are easily derived for the case of even (Y_c,Y_d) relative parity from the general form of the transition matrix given in Eq. (11).

$$I_0(W,\cos\theta)=|M_1|^2+|M_2|^2+2\sin^2\theta|M_3|^2+2\sin^2\theta|M_4|^2+|M_2+2(1+\cos\theta)M_5|^2 +|M_2+2(1-\cos\theta)M_6|^2+4\sin^2\theta[|M_7|^2+|M_8|^2], \quad (A1)$$

$$I_0\mathbf{P}_{\bar{c}}=I_0P_{\bar{c}}\hat{n}=2\hat{n}\sin\theta\{\mathrm{Re}[M_1^*(M_3+M_4)+M_2^*(M_3-M_4)] +4\,\mathrm{Im}[M_8^*M_2-(1+\cos\theta)(M_7-M_8)^*M_5+(1-\cos\theta)(M_7+M_8)^*M_6]\}, \quad (A2)$$

$$I_0\mathbf{P}_d=I_0P_d\hat{n}=2\hat{n}\sin\theta\{\mathrm{Re}[M_1^*(M_3-M_4)+M_2^*(M_3+M_4)] +4\,\mathrm{Im}[-M_8^*M_2-(1+\cos\theta)(M_7+M_8)^*M_5+(1-\cos\theta)(M_7-M_8)^*M_6]\}, \quad (A3)$$

$$I_0C_{ll}=2\,\mathrm{Re}[M_2^*(M_1-M_2)+2(1+\cos\theta)M_1^*M_5-2(1-\cos\theta)M_2^*M_6] -8\sin^2\theta\,\mathrm{Im}[M_3^*M_7-M_4^*M_8], \quad (A4)$$

$$I_0C_{mm}=2\,\mathrm{Re}[M_2^*(M_1-M_2)-2(1+\cos\theta)M_2^*M_5+2(1-\cos\theta)M_1^*M_6] +8\sin^2\theta\,\mathrm{Im}[M_3^*M_7+M_4^*M_8], \quad (A5)$$

[18] H. D. D. Watson, Nuovo Cimento **29**, 1338 (1963).
[19] M. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojeicki, Phys. Rev Letters **6**, 300 (1961).
[20] The correlation parameters quoted by Sopkovich refer to a different coordinate system than that used in the present paper.

$$I_0C_{nn}=2\,\mathrm{Re}[M_2^*(M_1-M_2)-2(1+\cos\theta)M_2^*M_5-2(1-\cos\theta)M_2^*M_6 -4\sin^2\theta M_5^*M_6]+4\sin^2\theta[|M_7|^2-|M_8|^2]\,, \quad \text{(A6)}$$

$$I_0C_{lm}=4\sin\theta\,\mathrm{Im}[(1+\cos\theta)(M_3-M_4)^*M_5-(1-\cos\theta)(M_3+M_4)^*M_6 +M_2^*M_4]+4\sin\theta\,\mathrm{Re}[M_1^*(M_7+M_8)+M_2^*(M_7-M_8)]\,, \quad \text{(A7)}$$

$$I_0C_{ml}=4\sin\theta\,\mathrm{Im}[(1+\cos\theta)(M_3+M_4)^*M_5-(1-\cos\theta)(M_3-M_4)^*M_6 -M_2^*M_4]+4\sin\theta\,\mathrm{Re}[M_2^*(M_7+M_8)+M_1^*(M_7-M_8)]\,, \quad \text{(A8)}$$

$$C_{ln}=C_{nl}=C_{mn}=C_{nm}=0\,. \quad \text{(A9)}$$

The symmetry properties of the polarization and spin correlation parameters noted in Sec. IIb, the properties of the M_j under charge conjugation given in Eqs. (12), and the angular factors which appear explicitly in the foregoing expressions, are easily checked when these relations are used.

The general expressions for the differential cross section, the polarization, and the spin correlation parameters for the reaction $\bar{N}_a+N_b\rightarrow\bar{Y}_c+Y_d$ may be derived for the case of odd (Y_c,Y_d) relative parity using the transition matrix given in Eq. (13):

$$I_0(W,\cos\theta)=4(1+\cos\theta)[|M_1|^2+|M_2|^2+|M_8|^2+\sin^2\theta|M_5|^2] +4(1-\cos\theta)[|M_3|^2+|M_4|^2+|M_6|^2+\sin^2\theta|M_7|^2]\,, \quad \text{(A10)}$$

$$I_0\mathbf{P}_{\bar{c}}=I_0P_{\bar{c}}\hat{n}=4\hat{n}\sin\theta\,\mathrm{Re}\{(M_1-M_2)^*[(1+\cos\theta)M_5+M_6]+(M_3-M_4)^*[(1-\cos\theta)M_7-M_8]\} +4\hat{n}\sin\theta\,\mathrm{Im}\{(M_1+M_2)^*(M_3+M_4)+[(1+\cos\theta)M_5-M_6]^*[(1-\cos\theta)M_7+M_8]\}\,, \quad \text{(A11)}$$

$$I_0\mathbf{P}_d=I_0P_d\hat{n}=4\hat{n}\sin\theta\,\mathrm{Re}\{(M_1+M_2)^*[(1+\cos\theta)M_5-M_6]+(M_3+M_4)^*[(1-\cos\theta)M_7+M_8]\} +4\hat{n}\sin\theta\,\mathrm{Im}\{(M_1-M_2)^*(M_3-M_4)+[(1+\cos\theta)M_5+M_6]^*[(1-\cos\theta)M_7-M_8]\}\,, \quad \text{(A12)}$$

$$I_0C_{ll}=4(1+\cos\theta)[|M_1|^2-|M_2|^2-|M_8|^2]+4\sin^2\theta(1-\cos\theta)|M_7|^2 +8(1-\cos\theta)\,\mathrm{Im}[(1+\cos\theta)M_3^*M_5+M_4^*M_6]\,, \quad \text{(A13)}$$

$$I_0C_{mm}=4(1-\cos\theta)[|M_3|^2-|M_4|^2-|M_6|^2]+4\sin^2\theta(1+\cos\theta)|M_5|^2 -8(1+\cos\theta)\,\mathrm{Im}[(1-\cos\theta)M_1^*M_7-M_2^*M_8]\,, \quad \text{(A14)}$$

$$I_0C_{nn}=8(1+\cos\theta)\,\mathrm{Im}[(1-\cos\theta)M_1^*M_7+M_2^*M_8)-8(1-\cos\theta)\,\mathrm{Im}[(1+\cos\theta)M_3^*M_5-M_4^*M_6]\,, \quad \text{(A15)}$$

$$I_0C_{lm}=4\sin\theta\,\mathrm{Re}\{(M_1+M_2)^*(M_3-M_4)-[(1+\cos\theta)M_5+M_6]^*[(1-\cos\theta)M_7+M_8]\} +4\sin\theta\,\mathrm{Im}\{(M_3+M_4)^*[(1-\cos\theta)M_7-M_8]-(M_1-M_2)^*[(1+\cos\theta)M_5-M_6]\}\,, \quad \text{(A16)}$$

$$I_0C_{ml}=4\sin\theta\,\mathrm{Re}\{(M_1-M_2)^*(M_3+M_4)-[(1+\cos\theta)M_5-M_6]^*[(1-\cos\theta)M_7-M_8]\} +4\sin\theta\,\mathrm{Im}\{(M_3-M_4)^*[(1-\cos\theta)M_7+M_8]-(M_1+M_2)^*[(1+\cos\theta)M_5+M_6]\}\,, \quad \text{(A17)}$$

$$C_{ln}=C_{nl}=C_{mn}=C_{nm}=0\,. \quad \text{(A18)}$$

The symmetry properties of the polarization and spin correlation parameters noted in Sec. IIb are again readily checked.

APPENDIX B: GENERAL THEORY OF THE REACTION $\bar{N}_a+\bar{N}_b\rightarrow\bar{Y}_c+Y_d$

In this Appendix, we will consider in greater detail the formal structure of the Wolfenstein-Ashkin spin transition matrix $M(W,\theta,\phi)$, in particular, the interpretations to be accorded this matrix in a relativistic theory, and the structure of the coefficient functions $M_j(W,\cos\theta)$ in terms of the partial-wave transition amplitudes. It will be convenient in this discussion to parametrize the relativistic scattering matrix using the helicity representation for angular momentum introduced by Jacob and Wick[8]; we shall rely heavily on results derived in their paper.

The reaction $\bar{N}_a+N_b\rightarrow\bar{Y}_c+Y_d$ can be described in complete generality in the center-of-mass frame in terms of a helicity transition matrix $f(W,\theta,\phi)$, the elements of which are the transition amplitudes in the helicity representation,

$$f_{\lambda_{\bar{c}}\lambda_d;\lambda_{\bar{a}}\lambda_b}(W,\theta,\phi)=(2\pi/ip)\langle\theta\phi\lambda_{\bar{c}}\lambda_d|S-1|00\lambda_{\bar{a}}\lambda_b\rangle\,. \quad \text{(B1)}$$

In this expression, $|\theta\phi\lambda_1\lambda_2\rangle$ is the plane-wave helicity state defined by Jacob and Wick, in which particle 1 moves in the θ, ϕ direction with helicity λ_1, and particle 2 moves in the opposite direction with helicity λ_2. The momentum of either incident particle in the center-of-mass frame will be denoted by p, and the total energy in that frame, by W. The initial state of the system can be described by a density matrix ρ in the helicity space; the final state is then described by the density matrix $f\rho f^\dagger$. The differential reaction cross section, polarizations, and helicity correlations can be calculated by standard methods. For example, for spin $-\frac{1}{2}$ particles with no initial polarization,

$$I_0(W,\cos\theta)=\tfrac{1}{4}\operatorname{Tr}ff^\dagger=\tfrac{1}{4}\sum_{[\lambda]}|f_{\lambda_{\bar c}\lambda_d;\lambda_{\bar a}\lambda_b}|^2. \quad \text{(B2)}$$

The plane-wave helicity states have the property that the helicities are unchanged by Lorentz transformations along the directions of motion of the particles, provided those directions of motion are not reversed. In particular, the helicity of a particle is preserved under the transformation to its rest system. Thus, the particles described by the helicity state $|\theta\phi\lambda_1\lambda_2\rangle$ have spin projections λ_1 along the θ, ϕ direction, and λ_2 along the $\pi-\theta$, $\pi+\phi$ direction, when observed in their respective rest systems as reached by a simple Lorentz transformation from the center-of-mass system. The indices on the helicity amplitudes in Eq. (B1) are therefore equivalent to ordinary spin indices in the particle rest systems, referred, however, to a different axis of quantization for each particle; and the transition matrix expressed in terms of helicities is completely equivalent to a transition matrix connecting proper spin states (spin states in the particle rest frames). The Wolfenstein-Ashkin M-matrix is obtained by re-expressing this result in terms of proper spin states quantized with respect to a common fixed coordinate system. Since the original description in terms of helicities was correct relativistically, it is clear that the M-matrix approach is also completely general and relativistically correct, provided that spin expectation values are referred always to the individual rest frames of the various particles.

For the construction of the coefficient functions $M_J(W,\cos\theta)$, it will be convenient to change from a plane-wave representation of the transition amplitudes to a representation in terms of the total angular momentum quantum number J, and the z projection of the angular momentum $J_z=M$. The necessary transformation[8] is provided by the representation coefficients for the rotation group,[21]

$$\langle\theta\phi\lambda_1\lambda_2|JM\lambda_1'\lambda_2'\rangle=[(2J+1)/4\pi]^{1/2}\times\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}D_{M,\lambda_1-\lambda_2}{}^{J*}(\phi,\theta,-\phi)\,, \quad \text{(B3)}$$

with

$$D_{\lambda\mu}{}^J(\alpha\beta\gamma)=e^{-i\alpha}d_{\lambda\mu}{}^J(\beta)e^{-i\gamma}.$$

This transformation leads to the result of Jacob and Wick,

$$f_{\lambda_{\bar c}\lambda_d;\lambda_{\bar a}\lambda_b}(W,\theta,\varphi)=(2ip)^{-1}\sum_J(2J+1)\times S_J(\lambda_{\bar c}\lambda_d;\lambda_{\bar a}\lambda_b)D_{\lambda\mu}{}^{J*}(\varphi,\theta,-\varphi)\,, \quad \text{(B4)}$$
$$\lambda=\lambda_{\bar a}-\lambda_b,\ \mu=\lambda_{\bar c}-\lambda_d.$$

The rotational invariance of the interactions implies that the S matrix connects only states of the same J and M, and that the elements of S are in fact independent of M. If parity is conserved in the reaction, the partial-wave matrix elements S_J transform according to the relation

$$S_J(-\lambda_{\bar c},-\lambda_d;-\lambda_{\bar a},-\lambda_b)=\eta_pS_J(\lambda_{\bar c}\lambda_d;\lambda_{\bar a}\lambda_b)\,, \quad \text{(B5)}$$

where η_p is equal to $+1$ (-1) if the relative intrinsic parity of the initial and final particles is even (odd). This result can be cast in a more useful form by introducing eigenstates of the parity operator

$$|JM\pm;\lambda_1\lambda_2\rangle=(1/\sqrt{2})[|JM\lambda_1\lambda_2\rangle\pm|JM,-\lambda_1,-\lambda_2\rangle],\ \lambda_1>0\,, \quad \text{(B6)}$$

with the transformation properties

$$P|JM\pm;\lambda_1\lambda_2\rangle=\pm\eta_{12}(-1)^{J-1}|JM\pm;\lambda_1\lambda_2\rangle\,. \quad \text{(B7)}$$

Here η_{12} is the relative parity factor for particles 1 and 2. Since the matrix S_J connects only states with the same parity, it has at most eight nonvanishing elements in this representation. Ordering the parity eigenstates as $|+;++\rangle$, $|+;+-\rangle$, $|-;+-\rangle$, $|-;++\rangle$, one easily obtains the most general forms for S_J consistent with the conservation of angular momentum and parity.

$$S_J=\begin{pmatrix} a_J & b_J{}^- & 0 & 0\\ b_J{}^+ & c_J & 0 & 0\\ 0 & 0 & d_J & e_J{}^-\\ 0 & 0 & e_J{}^+ & f_J\end{pmatrix},\quad \eta_p=+1\,, \quad \text{(B8)}$$

and

$$S_J=\begin{pmatrix} 0 & 0 & A_J{}^+ & B_J{}^+\\ 0 & 0 & C_J{}^+ & D_J{}^+\\ A_J{}^- & C_J{}^- & 0 & 0\\ B_J{}^- & D_J{}^- & 0 & 0\end{pmatrix},\quad \eta_p=-1\,, \quad \text{(B9)}$$

where $\eta_p=\eta_{\bar a b}{}^*\eta_{\bar c d}=+1(-1)$ for even (odd) relative (Y_c,Y_d) parity. For either case, the matrix elements $A^\pm$ are of the form $A^\pm=A\pm A'$. The elements of S_J are functions of W alone. Time reversal invariance does not add any new restrictions on the matrix elements, since the time reversed interaction $\bar Y_c+Y_d\to\bar N_a+N_b$ is not accessible to experiment.

The final symmetry which we shall impose on S_J is that of charge conjugation invariance. The application of C to an antiparticle-particle helicity state $|JM\lambda_{\bar 1}\lambda_2\rangle$ interchanges the roles of $\bar 1$ and 2, without changing the helicities. Since the antiparticle index is conventionally written first, this interchange induces the transformation

$$C|JM\lambda_{\bar 1}\lambda_2\rangle=(-1)^{J-1}|JM\lambda_{\bar 2}\lambda_1\rangle\,, \quad \text{(B10)}$$

where the phase factor is obtained using the methods of Jacob and Wick.[8] The corresponding properties of the parity eigenstates are easily deduced. However, it must be recalled that the antiparticle helicity index in Eq. (B6) is restricted to positive values. This restriction introduces an extra minus sign in the transformation of the state $|JM-;+-\rangle$. The imposition of charge-conjugation invariance leads to relations between the elements of the matrices S_J of Eqs. (B8) and (B9), and those of the corresponding matrices $S_J{}^c$ for the charge-

[21] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

conjugate reaction. All of the matrix elements of S_J and S_J^c are in fact equal with the exceptions of e_J and e_J' in the even parity case ($\eta_p=1$), and A_J, $A_J'C_J$, and C_J' in the odd parity case ($\eta_p=-1$), and these simply change sign, e.g., $e_J^c=-e_J$, etc.

Returning to the matrix notation, we can express the helicity transition matrix $\mathbf{f}(\theta,\phi)$ in terms of S_J as

$$\mathbf{f}(w,\theta,\phi)=(2ip)^{-1}\sum_{JM}(2J+1) \times U_{JM}^{\dagger}(\theta,\phi)S_J(W)U_{JM}(0,0)\,, \quad \text{(B11)}$$

where the unitary matrices $U_{JM}(\theta,\phi)$ connect the eigenstates of J, M, and parity with the plane wave helicity states. If the latter are written as column vectors in the order $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$, $U_{JM}(\theta\phi)$ is found to be

$$U_{J,M}(\theta,\phi)=\frac{1}{\sqrt{2}}\begin{bmatrix} D_{M0}{}^J & 0 & 0 & D_{M0}{}^J \\ 0 & D_{M1}{}^J & D_{M,-1}{}^J & 0 \\ 0 & D_{M1}{}^J & D_{M,-1}{}^J & 0 \\ D_{M0}{}^J & 0 & 0 & -D_{M0}{}^J \end{bmatrix}. \quad \text{(B12)}$$

The rotation coefficients $D_{\lambda\mu}{}^J(\phi,\theta,-\phi)$ appear with the indicated arguments; in the special case $\theta=\phi=0$, $D_{\lambda\mu}{}^J(0,0,0)=\delta_{\lambda\mu}$. As noted previously, $\mathbf{f}(W,\theta,\phi)$ can be interpreted as a transition matrix which connects initial and final proper spin states with the quantization axis for each particle chosen along its direction of motion in the over-all center-of-mass system. The Wolfenstein-Ashkin spin transition matrix $M(W,\theta,\phi)$ is obtained from $\mathbf{f}(W,\theta,\phi)$ by re-expressing this matrix in a representation in which the proper spin states are quantized relative to a common axis. The necessary transformation is easily derived. We will denote by $|\lambda_1\lambda_2\rangle$ a two-particle spin state in which the spin components of particles 1 and 2 along the z axis (the direction of motion of the incident antiparticle) are equal in their respective rest systems to λ_1 and λ_2. In the initial helicity state $|00\lambda_{\bar{a}}\lambda_b\rangle$ in Eq. (B1), the antinucleon is assumed to move in the positive z direction, and the nucleon, in the negative z direction. Noting the phase conventions used by Jacob and Wick,[8] this state is seen to correspond to the spin state

$$(-1)^{\frac{1}{2}-\lambda_b}e^{-i\pi S_{b,y}}|\lambda_{\bar{a}}\lambda_b\rangle\,.$$

Similarly, the final helicity state $|\theta\phi\lambda_{\bar{c}}\lambda_d\rangle$, in which the antihyperon moves in the θ,ϕ direction, and the hyperon, in the $\pi-\theta$, $\pi+\phi$ direction, corresponds to a spin state

$$(-1)^{\frac{1}{2}-\lambda_d}\exp(-i\theta\hat{n}\cdot\mathbf{S})\exp(-i\pi S_{d,y})|\lambda_{\bar{c}}\lambda_d\rangle\,, \quad \mathbf{S}=\mathbf{S}_{\bar{c}}+\mathbf{S}_d.$$

In these expressions, $\mathbf{S}_j$ is the spin operator of particle j in its rest system, and the azimuthal orientation of the y axis is arbitrary. With these conventions, the spin transition matrix M is given by

$$M(W,\theta,\phi)=(-1)^{\frac{1}{2}-\lambda_b}(-1)^{\frac{1}{2}-\lambda_d}\exp-i\theta\hat{n}\cdot\mathbf{S} \times\exp(-i\pi S_{d,y})\mathbf{f}(W,\theta,\phi)\exp i\pi S_{b,y}. \quad \text{(B13)}$$

The 4×4 matrix M can be expressed as a linear combination of the sixteen independent matrices σ_{ij} defined in Sec. II a; the necessary relations are given in Eqs. (5). We will use the convention that the Pauli matrices $\boldsymbol{\sigma}_1$ act in the 2×2 proper spin space of the antiparticles, while the matrices $\boldsymbol{\sigma}_2$ act in the spin space of the particles. The result for M in Eq. (B13) may then be rewritten in terms of the Pauli matrices as

$$M(W,\theta,\phi)=\exp[-\tfrac{1}{2}\theta\hat{n}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)]\sigma_{2x}\mathbf{f}(W,\theta,\phi)\sigma_{2x}. \quad \text{(B14)}$$

After a lengthy but simple calculation using the expression for $\mathbf{f}$ given in Eq. (B12), and the matrix representations for U_{JM} and S_J, the transition matrices for even and odd relative (Y_a,Y_b) parity may be reduced to the forms given in Eqs. (11) and (13). The resulting coefficient functions $M_j(W,\cos\theta)$ are given in the even parity case by

$$M_1=(16ip)^{-1}\sum_J(2J+1)[2a_J\cos\theta d_{00}{}^J+2f_Jd_{00}{}^J-4b_J\sin\theta d_{10}{}^J +(c_J+d_J)(1+\cos\theta)d_{11}{}^J-(c_J-d_J)(1-\cos\theta)d_{-1,1}{}^J]\,, \quad \text{(B15)}$$

$$M_2=(16ip)^{-1}\sum_J(2J+1)[2a_J\cos\theta d_{00}{}^J-2f_Jd_{00}{}^J-4b_J\sin\theta d_{10}{}^J -(c_J+d_J)(1-\cos\theta)d_{11}{}^J+(c_J-d_J)(1+\cos\theta)d_{-1,1}{}^J]\,, \quad \text{(B16)}$$

$$\sin\theta M_3=-i(16ip)^{-1}\sum_J(2J+1)[2a_J\sin\theta d_{00}{}^J+4b_J\cos\theta d_{10}{}^J +(c_J+d_J)\sin\theta d_{11}{}^J+(c_J-d_J)\sin\theta d_{-1,1}{}^J]\,, \quad \text{(B17)}$$

$$\sin\theta M_4=i(4ip)^{-1}\sum_J(2J+1)e_Jd_{10}{}^J\,, \quad \text{(B18)}$$

$$2(1+\cos\theta)M_5=-(16ip)^{-1}\sum_J(2J+1)[2a_J(1+\cos\theta)d_{00}{}^J-4b_J\sin\theta d_{10}{}^J -(c_J+d_J)(3-\cos\theta)d_{11}{}^J+(c_J-d_J)(1+\cos\theta)d_{-1,1}{}^J]\,, \quad \text{(B19)}$$

$$2(1-\cos\theta)M_6=(16ip)^{-1}\sum_J(2J+1)[2a_J(1-\cos\theta)d_{00}{}^J+4b_J\sin\theta d_{10}{}^J +(c_J+d_J)(1-\cos\theta)d_{11}{}^J-(c_J-d_J)(3+\cos\theta)d_{-1,1}{}^J]\,, \quad \text{(B20)}$$

$$2\sin\theta M_7=-(4ip)^{-1}\sum_J(2J+1)b_J'd_{10}{}^J\,, \quad \text{(B21)}$$

$$2\sin\theta M_8=(4ip)^{-1}\sum_J(2J+1)e_J'd_{10}{}^J. \quad \text{(B22)}$$

The rotation coefficients $d_{\lambda\mu}{}^J(\theta)$ for the indices of interest are given by

$$d_{00}{}^J(\theta)=P_J(\cos\theta)\,, \tag{B23}$$

$$d_{10}{}^J(\theta)=-\frac{\sin\theta}{[J(J+1)]^{1/2}}\frac{d}{d(\cos\theta)}P_J(\cos\theta)\,, \tag{B24}$$

$$d_{-1,1}{}^J(\theta)=\frac{1-\cos\theta}{J(J+1)}\left[\frac{d}{d(\cos\theta)}P_J(\cos\theta)+(1+\cos\theta)\frac{d^2}{d(\cos\theta)^2}P_J(\cos\theta)\right], \tag{B25}$$

$$d_{11}{}^J(\theta)=\frac{1+\cos\theta}{J(J+1)}\left[\frac{d}{d(\cos\theta)}P_J(\cos\theta)-(1-\cos\theta)\frac{d^2}{d(\cos\theta)^2}P_J(\cos\theta)\right], \tag{B26}$$

where $P_J(\cos\theta)$ is the ordinary Legendre polynomial. Using these results, it is readily verified that the angular factors which appear on the left-hand sides of Eqs. (B17)–(B22) are contained also on the right-hand sides. Since these are precisely the factors contained in the vectors **l**, **m**, and **n**, the validity of the result for M given in Eq. (11) is established. In particular, the coefficient functions $M_j(W,\cos\theta)$ are nonsingular for $|\cos\theta|\leq 1$ and we may consequently use the angular dependence of **l**, **m**, and **n** to obtain some information about the angular variation of the reaction cross section and the polarization and spin correlation parameters. The apparent asymmetry of the result for M, in which $\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}\cdot\mathbf{l}$ and $\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{m}$ appear explicitly, but in which $\boldsymbol{\sigma}\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}$ is absent, is connected with the presence of the term $M_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. Although $M_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ can be expressed as a linear combination of the foregoing terms, the coefficients are, in general, singular at $\theta=0$ or $\theta=\pi$, and it is preferable to use $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ as one of the independent invariants. However, it may be useful in some situations to introduce an extra term in Eq. (11), and use an overcomplete set of matrices containing $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{l}$, $\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{m}$, and $\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}$. The coefficient functions, only three of which are independent, can be so chosen that the coefficient functions are nonsingular.

A similar calculation for the case of odd (Y_a,Y_b) relative parity leads to a spin transition matrix of the form given in Eq. (13), with coefficient functions $M_j(W,\cos\theta)$ expressed in terms of the elements of S_J, Eq. (B9), as

$$2\cos\tfrac{1}{2}\theta M_1 =-(4ip)^{-1}\textstyle\sum_J(2J+1) \times[A_J\sin\tfrac{1}{2}\theta d_{10}{}^J-C_J\cos\tfrac{1}{2}\theta d_{11}{}^J]\,, \tag{B27}$$

$$2\cos\tfrac{1}{2}\theta M_2 =(4ip)^{-1}\textstyle\sum_J(2J+1) \times[B_J\cos\tfrac{1}{2}\theta d_{00}{}^J-D_J\sin\tfrac{1}{2}\theta d_{10}{}^J]\,, \tag{B28}$$

$$2\sin\tfrac{1}{2}\theta M_3 =(4ip)^{-1}\textstyle\sum_J(2J+1) \times[A_J'\cos\tfrac{1}{2}\theta d_{10}{}^J-C_J'\sin\tfrac{1}{2}\theta d_{-1,1}{}^J]\,, \tag{B29}$$

$$2\sin\tfrac{1}{2}\theta M_4 =(4ip)^{-1}\textstyle\sum_J(2J+1) \times[B_J'\sin\tfrac{1}{2}\theta d_{00}{}^J+D_J'\cos\tfrac{1}{2}\theta d_{10}{}^J]\,, \tag{B30}$$

$$2\cos\tfrac{1}{2}\theta\sin\theta M_5 =-i(4ip)^{-1}\textstyle\sum_J(2J+1) \times[A_J\cos\tfrac{1}{2}\theta d_{10}{}^J+C_J\sin\tfrac{1}{2}\theta d_{11}{}^J]\,, \tag{B31}$$

$$2\sin\tfrac{1}{2}\theta M_6 =i(4ip)^{-1}\textstyle\sum_J(2J+1) \times[B_J\sin\tfrac{1}{2}\theta d_{00}{}^J+D_J\cos\tfrac{1}{2}\theta d_{10}{}^J]\,, \tag{B32}$$

$$2\sin\tfrac{1}{2}\theta\sin\theta M_7 =-i(4ip)^{-1}\textstyle\sum_J(2J+1) \times[A_J'\sin\tfrac{1}{2}\theta d_{10}{}^J+C_J'\cos\tfrac{1}{2}\theta d_{-1,1}{}^J]\,, \tag{B33}$$

$$2\cos\tfrac{1}{2}\theta M_8 =i(4ip)^{-1}\textstyle\sum_J(2J+1) \times[B_J'\cos\tfrac{1}{2}\theta d_{00}{}^J-D_J'\sin\tfrac{1}{2}\theta d_{10}{}^J]\,. \tag{B34}$$

As before, one can easily verify that the indicated angular factors appear on the right-hand sides of these expressions, so that the functions M_j are nonsingular for $|\cos\theta|\leq 1$. It is in fact for this reason that we have chosen to use the invariants $\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{l}$ and $\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{m}$ in Eq. (13) rather than $[\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{n}-\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{m}]$ and $[\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{l}-\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{n}]$; the latter would be equivalent to the former as far as their spin dependence is concerned, but would require coefficients which diverged for $\theta\to 0$ and $\theta\to\pi$, respectively. In the even parity case, the forms $\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\cdot\mathbf{n}$ and $[\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\mathbf{m}-\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{l}]$ are completely equivalent. These results, and also the difficulty with the invariants $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{l}$, $\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{m}$, and $\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}$ in the case of even (Y_c,Y_d) relative parity, are special instances of a rule, that the minimal angular dependence of the coefficient functions is that of the vectors **l**, **m**, and **n** in the least complex set of invariants, that is, the set which contains the least number of factors of $\hat{l}$, $\hat{m}$, or $\hat{n}$ in each invariant. This rule is also valid for reactions involving only spin -0 and spin $-\frac{1}{2}$ particles, and is probably true in general.